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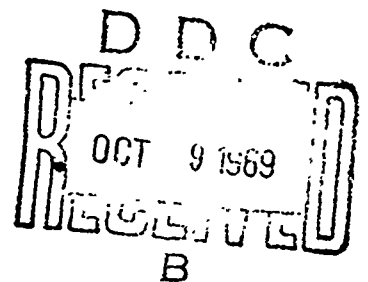
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SYSTEM DESIGN CONSIDERATIONS FOR A BAYESIAN ANTISUBMARINE WARFARE (ASW) INFORMATION PROCESSOR

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PRC R-1319

April 1969

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PLANNING RESEARCH CORPORATION
LOS ANGELES, CALIFORNIA   WASHINGTON, D.C.

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SYSTEM DESIGN CONSIDERATIONS FOR A
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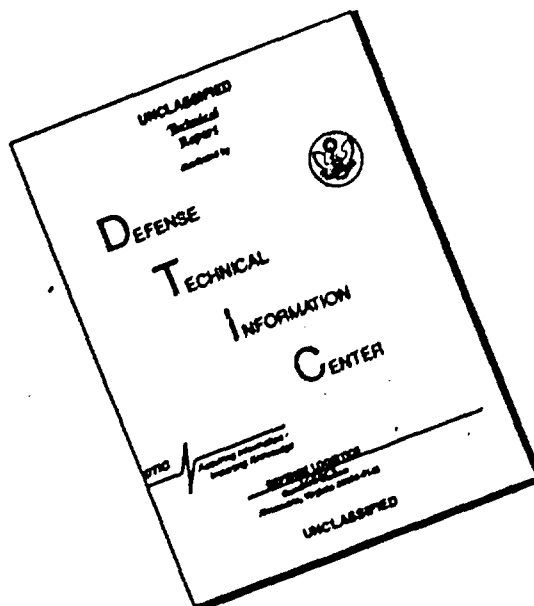PRC R-1319

April 1969

Prepared for

Office of Naval Research
Under Contract Nonr 4994(00)

By

Livingston Dodson

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ABSTRACT

This study continues the investigation of methods for applying Bayesian statistics to the problem of locating submarines from data made available to an ASW Command Center. The work performed during earlier phases of this project is documented in PRC reports: R-808, Bayesian Information Modeling in ASW, Feb. 1966; R-1001, Application of Bayes' Theorem to Predicting Submarine Locations (U), Feb. 1967 (Secret); and R-1133, Analysis of ASW Data Using Bayesian Techniques (U), Feb. 1968 (Secret).

In particular, this study investigates methods for:

- The simultaneous integration of data across an area containing possibly more than one, i. e. K, submarines.

- Determination of "probable tracks" and their "validity" in conjunction with processes which determine a submarine "density map" for display purposes.

- A generalization of the previous concept of submarine "diffusion" (to represent unknown but possible submarine movements) in order to bring the model closer to the complexity of real operations.

- A measurement of effectiveness of any combined ASW system,i. e. , sensors plus data integration, concerning its localization of enemy submarines.

PREFACE

This report presents the latest findings under a series of con-
tracts for the Office of Naval Research (ONR) to investigate applica-
tions of Bayesian techniques to the Antisubmarine Warfare (ASW) infor-
mation processing function.   Because the work reported in this docu-
ment is founded on the previous research carried out by Planning Re-
search Corporation (PRC), the following paragraphs present summaries
of the earlier studies and demonstrate how each utilizes and extends all
previous work.   Finally, the research topic for the present study is
identified, and the ground rules adopted for its development are
presented.

A.   Sub-Barrier Situation (PRC Report Number R-808)

The credibility and tentative feasibility of using Bayesian tech-
niques to model integration of ASW information and data were initially
demonstrated in a study of a sub-barrier exercise.   Several kinds of
information were identified in this study as describing the dynamic in-
formation environment of ASW operations:

- Current situation estimates
- Descriptions of past (transpired) events which are
  pertinent to the estimate
- Revised current situation estimates

A dynamic process was defined which used these three kinds of infor-
mation.   As descriptions of transpired events were received by the
command function, they were used to transform the current situation
estimate into a revised current situation estimate.   Bayesian techniques
were the basis for this transformation.   Decisionmaking was not modeled
but is related to the process, in that decisions are based on the current
situation estimate.

One particular situation estimate was distinguished.   The first
situation estimate, the basis for subsequent revision, was taken as an
assumption.   Other assumptions involved the parameters of sensor

performance. A third, and less obvious, assumption was that the portion of the real world quantified into the model was sufficiently large to allow meaningful results to be obtained.

The application of the model yielded encouraging results. The model provided a current situation estimate for each moment of time through the duration of the exercise. Had the commander of the ASW forces used the situation estimates generated by the model, he would have made decisions very similar to the actual decisions of the exercise, as evidenced by his utilization of ASW resources during the exercise. Further, the model was applied in a reverse fashion to obtain a characterization of the initial situation estimate employed by the commander (an assumption in the first application of the model). The results corresponded closely with what the commander probably should have known at the beginning of the exercise, insofar as that was reconstructable from the exercise report.

The qualitative and quantitative agreement between the actual exercise and the model results demonstrated tentative feasibility of using Bayesian methods for modeling the information environment and processes of ASW operations. Two other results of this research were that (1) a useful classification of the kinds of information prevalent in a dynamic ASW command function (at any level) was developed; (2) a substantial amount of analysis was executed, ensuring a sound theoretical base for the application of Bayesian techniques.

B. Oceanwide Situation (PRC R-1001)

The next research excursion involved a more complex application of Bayesian techniques to modeling the information environment associated with ASW operations. It was recognized that the sub-barrier situation had several analytical limitations:

- Single dimension
- Goal-oriented submarine
- Operationally homogeneous exercise
- Mathematically continuous situation estimate
- Single submarine

The sub-barrier case was treated as a geographically one-dimensional case. It was therefore desirable to investigate modeling a geographically two-dimensional case. The first application involved only sonobuoys emplaced and monitored by VP aircraft. It was therefore desirable to answer the question of whether the Bayesian modeling approach could be validly and credibly applied to integrating information received from a variety of sensors. The first application involved a strongly goal-oriented submarine, that is, one which was carrying out a mission that left it little freedom to take evasive action as a tactic to counter the ASW operations. This fact was used in the modeling and, in part, allowed the simplification to single-dimensioned analysis. It was therefore highly desirable that the next research excursion investigate modeling of less goal-oriented submarine operations. Further, the first model treated the situation estimate as a continuous probability density function of the submarine's location. For a number of reasons it was desirable to investigate discrete methods of representing situation estimates. Among these reasons was the susceptibility of discrete representations to shorter computational procedures and ease of display. Finally, the first model dealt with a single submarine, and it was desirable to know how well Bayesian techniques would perform in a model which handled more than one submarine.

The second research excursion comprised, therefore, research for and development of a model which was applied to a more extensive ASW exercise and an evaluation of that application. The ASW exercise which was studied covered a large expanse of the Pacific Ocean. It involved several submarines and a nonhomogeneous force of ASW platforms and sensors. The submarines had sufficient flexibility in their missions so as not to exhibit highly goal-oriented behavior; and the geographic area was two-dimensional and large, forcing use of discrete representations of situation estimates.

The results of this second research effort and model application demonstrated further feasibility of application of Bayesian techniques. More specifically, the following results, among others, were obtained:

- Feasible methods for combining data from various kinds of sensors and platforms
- Feasible methods for handling a two-dimensional situation
- Credible sequence of situation estimates based on data from a large and complex exercise
- Description of the kinds of data needed for adequate application of Bayesian techniques
- Analytical methods for handling probabilistic dependencies in certain kinds of data
- Analytical methods for conversion of sensor detection data to forms amenable to analysis by Bayesian techniques
- Mathematical approximations allowing efficient computation of Bayesian statistics

In general, then, this second research effort showed that Bayes' Theorem and the substantial analytical structure that was developed to support it formed an appropriate and feasible tool for modeling the dynamic information environment associated with threat estimate in ASW.

C.   Analytical Refinements (PRC R-1133)

The two studies described above yielded useful results, but did not exhaust the need for research into application of Bayesian techniques in understanding and structuring ASW data. Specifically, research conducted over the period covered by the next effort was aimed at satisfying these further objectives:

- Application of Bayesian modeling techniques in a more true-to-life situation
- Testing of new extensions of Bayesian techniques
- Development of methods for handling probabilistically dependent datums
- Investigation of the sensitivity of the ASW information environment to precision of sensor performance parameters
- Refinement of computational procedures

The first of these objectives, the need for application in a situation more closely approximating real life, had several subobjectives. The sensor detection data should be similar in form and content to that actually used by the component of the ASW force whose information integration function was being modeled. Further, the geographical area of interest should correspond to the area held in interest by that component. Finally, there was a general need for more exposure to operational ASW personnel in order to gain a clearer understanding of the information environment in which they operate.

The third objective listed above also deserves comment. Associated with the oceanwide situation was development of procedures for handling certain kinds of probabilistic dependence. This methodology was limited to dependencies within one datum (observation). Cases of probabilistic dependence among several simultaneous or sequential observations were still not handled analytically. It was the objective, then, to develop ways of handling these kinds of observations within the framework of a Bayesian modeling system. Once this capability was implemented, it would then be possible to test the sensitivity of the model results to degree of dependency.

A new model design was undertaken. Development of this design allowed consolidation of experience accumulated from using the previous two models and allowed testing of new Bayesian techniques, development of more efficient computational procedures, and development of methods for handling certain probabilistic dependencies. Further, there was an opportunity to work with personnel of the Commander ASW Forces Pacific (COMASWFORPAC). Development of a new model design provided more opportunities to benefit from their participation. The design activity itself consisted of several substantial analysis tasks, the product of which was a model that could operate in either of two modes, called "diffusion" and "track-analysis."

The computer model included a treatment of both time-simultaneous and time-sequential dependencies among collected data, a problem which heretofore had not been amenable to practical analytic solution. Second, it incorporated a unique and more powerful way to apply

Bayes' Theorem. This new manner of application, called track-analysis, allowed consideration of time-sequential dependencies and the geometry of submarine tracks relative to the geometry of contours of sensor detection probabilities. Track analysis also provided a framework within which a number of other considerations might be potentially included.

D.   ASW Information Processor (PRC R-1319)

The current research effort has been directed toward development of an ASW information processor that embodies all of the research results in this series of studies and integrates multisource information inputs to develop situation displays. In presenting the system design for an ASW information processor in this document, every effort has been made to avoid a detailed mathematical development, and most of the burden of the description is carried by a verbal development of the system concepts. Several reasons for this are of interest:

1.   The mathematical theorems called upon in the system are of no interest in themselves, since all have the properties of being simple and long-established.

2.   The ASW information system problems are solved and the difficulties overcome by new concepts in applying this rudimentary mathematics.

3.   The complexity of the system description is due solely to the interrelationships among a sizable number of such concepts.

4.   An exhaustive mathematical treatment would have tended to obscure the meaning of what is, after all, an interrelated set of inherently simple individual ideas.

5.   The system design is capable of being applied to any level of command operating across an area of any size, from an individual ship or aircraft to a single worldwide or oceanwide command. Some of the system parameters take on a specific meaning and value only when the level and scale of the command are fixed for a given application. Therefore, a "system" in the applied sense cannot be exhaustively described in the terms of this document except at the point of actual application.

6.   While the concepts and equations are simple, the computational burden is great, and a digital computer is required for even the smallest application. In a specific application of the system, computer time and storage constraints should be considered along with the other variables when fixing some of the system parameters, such as "cell size." This aspect of the problem has been ignored   again, so as not to obscure the description of essential concepts. For the same reason, although the system calls for certain "displays" of density (or contour) maps and networks of probable tracks, no consideration is given to the hardware aspects of this problem.

I. A GENERAL DEFINITION OF AN ASW INFORMATION PROCESSING SYSTEM (ASWIPS)

In designing a military Command and Control system (CAC) method-
ology, or treating any one of its numerous subsidiary problems, a sharp
definition of the end product (or output) desired is helpful at all stages of
progress. Even better would be a numerical measure which could be
used to: (1) predict the difference in effectiveness between two or more
potential CAC methods or submethods, to assist in making design choices
during a developmental phase; and (2) assess the status of an operational
CAC in real time in relation to the desired or practically achievable
status.

Unfortunately for this desirable aim, it turns out upon investigation
...at most CAC's have not one but many purposes, end uses, and possible
outputs. Certainly the current and future planned ASWCAC's are no
exception to this. For example, some of the end uses of an ASWCAC
include:

- Allocation of forces for the purpose of detection, localization,
  and/or destruction of enemy submarines
- Convoy planning and real time guidance
- Warning to other U.S. command systems

Obviously it would be undesirable (and perhaps impossible) to
create a rigid set of measures in advance for these various end uses,
some of which may not be foreseen but created under the pressure of
immediate need. The end use of a command system is therefore best
left undefined; that is, left to the discretion of the command and his staff
from moment to moment.

What in this case can be done by way of preformulation of proced-
ures to assist the command without at the same time limiting or inter-
fering with the performance of his mission? If we determine that all of
the possible end uses of the system are predicated upon the ability of
the ASWCAC to receive, process, and display information, then we are
in a position to make a fundamental bifurcation of an ASWCAC's uses,
methods, and subsidiary methods into:

(1)   Those concerned with the transmission, integration, and display of information.

(2)   Those concerned with the determination of actions.

The actions selected are usually based upon the output of procedures from the first category, but only in part. Other considerations may involve current needs, aims, and even information made available outside the procedures of category (1). We will, then, modify the originally stated desire for a sharp definition of an ASWCAC system output by limiting the definition of the "system" under discussion to the subsystem of category (1), which will be referred to as an ASW Information Processing System (ASWIPS).

"Information," as such, is amenable to quantification independent of the possible uses to which it may be put, and may properly become the basis of a measure of effectiveness of an ASWIPS. Without formulating too technical a definition of "information," we can describe it by the following remarks.

"Information" is a relative, not an absolute, concept. To speak of a given quantity of "information" is meaningless without an understanding of the question to which the "information" is relevant. This characteristic is also shared by the concept of "probability." It is meaningless to speak of the probability of an event unless there is at least an implied reference event (such as a "trial"). In this sense, all "probabilities" are conditional--conditional upon the certain occurrence of the reference event.[1]

Formulation of a measure of "information," therefore, requires a prior formulation of the reference problem. By its very nature, this reference problem can always be expressed as a question. The quantity used as a measure should then indicate the closeness of one's ability to answer the reference question.

---

[1] A topical example of the importance of forgetting that the "reference event" is an integral part of the concept of "probability" is the meaninglessness inherent in the question phrased as, "What is the probability of life on other planets?" The reply to this question should properly be, "Relative to what events known for certain?" If the answer is, "Relative to our knowledge of life on planets examined in detail so far," then the number of such trials equals 1, our Earth experience. This yields $p = 1$ to the original probability in question, which is, of course, patently absurd.

What is the nature of such "reference questions?" Evidently they are, in the same spirit as the concept of "probability, " questions about the occurrence of "events. " Thus we can further specify that the information measure must be defined cn the basis of a question about the occurrence of a specified event. If the event is redefined, so is the measure.

"Information" is something different from "data. " A given amount of "data" may or may not contain "information" relevant to a given problem. Similarly, a certain piece of data may contain no information relative to one problem but carry valuable information relative to a different problem. What is a "signal" to one engineer may be "noise" to another.

"Data" can be more precisely defined by considering it to be a collection of statements about the occurrence of events. A "datum" would be a statement that one particular event has occurred. The event which has occurred may have a variable relationship with the event of the reference question, or it may be the same event as in the reference question.

At this point, a distinction can be made between two subsystems always involved in a CAC. One type, which might be called "data producers, " has the function of determining the occurrence of certain events and forwarding "statements" about these to the second subsystem. The other which may be called "data integrators, " has the function of determining the "impact" of the data on the possibility of the occurrence of the event in the reference question. It can be seen that the quantity of data available (the number of possibly associated events known to have occurred), as well as the quality of this data (the degree of relationship between the observed events and the reference event) and the astuteness of the data integrator, will have an effect on the amount of "information" output.

The functions associated with the "data integrator" in ASW are those that will be referred to by ASWIPS. The approach to be used is as follows:

(1)   Develop a quantitative measure of information relevant to
the ASW localization problem which may be used to assess
the outputs of any competing candidate ASWIPS methods
(either conceptually in advance by mathematical arguments
or empirically). The input "data" available to these differ-
ent ASWIPS from "data producers" must be identical, to
avoid biasing the conclusions.

(2)   Formulate at least one such ASWIPS method and show by
a priori arguments that it is a good approach in terms of
the above measure.

(3)   Include in the ASWIPS method of step (2) a method for
estimating the information measure in real time. This
enables an assessment of the status of the ASWCAC (data
producers plus data integrator) as a whole during operations.

In subsequent development, every attempt will be made to keep
the ASWIPS methods to be investigated as generalized as possible and
applicable to any of the several command levels currently in operation,
from an individual ship or aircraft to a HUK force command, a sector
command, or ASWFORLANT/PAC.

II. A MEASURE OF EFFECTIVENESS

In Section I the conclusion was reached that the proper measure of effectiveness for an ASWIPS is an "information" measure. It was also pointed out that any single measure of information is meaningful only in relation to a single corresponding question about the occurrence of a "reference event." This section discusses what the reference question and event are or should be for any ASWIPS.

A. Information Measures

Evidently the reference question is something similar to, "Where are the enemy submarines?" We can find the reference event implied in this question by considering what category of statement would be an absolutely complete answer to "Where are the submarines?" This would seem to be a list of spatial coordinates, one for each submarine, at a given time. The "reference event" is an enemy submarine at latitude ___, longitude ___, at time T.

To make the question more reasonable and flexible, suppose that the ASW command being considered has responsibility for ASW operations across a given sector with area A. Suppose further that this sector is subdivided into N equal area cells, such that complete satisfaction for all purposes is reached if it is known with certainty that a given submarine is within a given cell. That is, there is no need for localization beyond this degree of accuracy. Let this type of cell be called a type B cell with area

$$b = \frac{A}{N} \tag{1}$$

Let K be the best estimate of the number of enemy submarines in the sector of area A at a given time. Identify the individual cells by an index i = 1, 2, ...N.

The answer of a given ASWIPS at a fixed time to the reference question can then be described as a list of expected submarine densities, d_i, for i = 1, 2, ...N. Our measuring problem is confined to

describing the "adequacy" of this list, or the density distribution.

It should be noted first, however, that since the N cells are mutually exclusive and exhaustive with respect to each of the K submarines, there are the implied constraints

$$\sum_{i=1}^{N} d_i = K \qquad (2)$$

$$d_i \leq 0 \quad \text{for } i = 1, 2, \ldots N \qquad (3)$$

If this distribution is converted to a probability density distribution by dividing each expected number of submarines by K, as

$$P_1 = \frac{d_i}{K} \qquad (4)$$

which yields the constraints

$$\sum_{i=1}^{N} P_i = 1 \qquad (5)$$

$$p_i \geq 0 \quad \text{for } i = 1, 2, \ldots N \qquad (6)$$

then the discussion of possible ways to measure the "information" content of the latter distribution can begin with Shannon and Weaver's[1] negative entropy measure. This now classical and widely discussed formulation is defined as

$$H = -\sum_{i=1}^{N} P_i \log_2 P_i \qquad (7)$$

---

[1] C. E. Shannon and W. Weaver, The Mathematical Theory of Communication (University of Illinois Press, Urbana, 1949).

The measure H is called the "negative entropy" of the probabi- lity density function, p_i . It is, however, a positive quantity due to the negative sign preceding the summation. Notice that the logarithm is to the base 2. While the base is arbitrary in fact, the base 2 is selected for defining H primarily because the binary system is so widely appli- cable in communications, Shannon's original field of application.

H is expressed in units called "bits." One "bit," the unit of information, may be defined as that quantity of information necessary to reduce the number of equally probable alternatives by a factor of 2.

For example, suppose there is one submarine located somewhere within a square sector subdivided into 100 cells. If nothing other than this is known, then each cell may be considered equally likely to contain the submarine; each cell has probability $1/N = 1/100$. If a report is then received specifying that the submarine is actually somewhere within, say, the northern half of the sector, then the uncertainty has been reduced by a factor of 2, since we now have 50 cells equally likely to contain the submarine. This message transmitted one "bit" of infor- mation. Similarly, the measure H for the two distributions just des- cribed would differ by one "bit."

H is constrained between the limits

$$0 \geq H \geq -\log \frac{1}{N} \qquad (8)$$

where a score of zero represents perfect or maximum information. This occurs only if $p = 1$ for some i and $p = 0$ for all others. On the other hand, minimum information occurs in the completely random case in which $p_i = 1/N$ and $H = -\log 1/N$.

If we apply H as a measure to locating K > 1 submarines within a sector of area A with N cells of area, then the upper limit on H remains $-\log 1/N$, but the lower limit is partly a function of K and partly a function of the number of cells the perfectly located submar- ines happen to occupy. Since this number of cells may vary from 1

to K cells, we have as constraints for $K > 1$

$$H \leq -\log \frac{1}{N} \qquad (9)$$

and perfect information is then implied if

$$0 \leq H \leq -\log \frac{1}{K} \qquad (10)$$

The measure H has several properties which make it a desirable candidate for the "information" measure sought:

- It is currently used and understood in many other application areas.
- It is easily computed.
- Its minimum and maximum values are easily interpretable in terms of the ASW problem.
- Between these limits it varies linearly with "information" available.

Its chief drawback is that, for values between the limits, the unit of "bits" is not intuitively meaningful or even applicable to the ASW search and localization problem. This disadvantage would not be eliminated by simply changing the base of the logarithm. A unit of measure related to area (or number of cells) searched or to be searched would be the most easily interpreted type of unit.

To make this notion more precise, let us attempt to formulate an information measure in terms of "the expected minimal number of cells which contain all K submarines." This can best be explained by a numerical example, and the following figure with $N = 4$, $K = 2$, shows example submarine densities, d_i for $i = 1, 2, 3, 4$.

The corresponding probability density function would be:

Cell No.
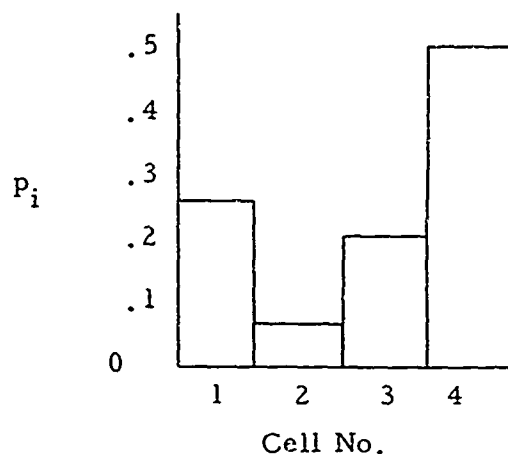
Now suppose that the cell probabilities are rearranged in rank order, with the greatest probability ranked 1 and the rest in descending fashion to give the graph below.

Cell Rank
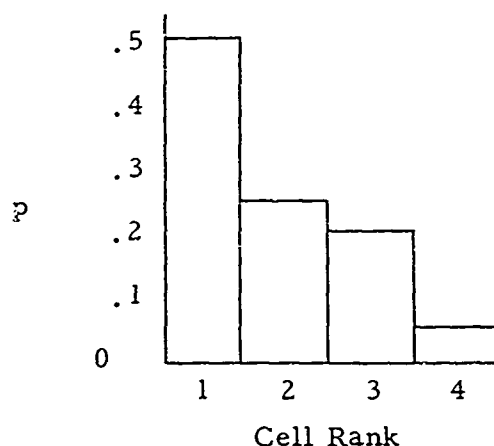
The mean of this density function is the measure sought.

An intuitive approach to the concept is to imagine searching the cells in sequence by rank order. The expected (mean) number of cells searched before finding all K submarines is the mean of the above distribution. Since the imaginary search is conducted in an optimal sequence, the resulting mean is the "minimum" across all possible sequences.

The maximum value for M , that for the case of equally likely cells, is N/2 . The minimum value, that for the case where all K submarines are located within K or less cells (say μ), is $\mu/2$, hence the constraints.

$$\frac{\mu}{2} \leq M \leq \frac{N}{2}$$

$$\frac{1}{2} \leq \frac{\mu}{2} \leq \frac{K}{2}$$

Notice that μ is controlled by the enemy submarines, not by the amount of information available.

We can establish the functional relationship between H and M as follows. Let $\overline{H}$ and $\overline{M}$ represent the maximum values of the two measures. Since each bit subtracted from $\overline{H}$ represents a halving of the amount of uncertainty implied by the probability density function p_i , there is corresponding halving of the expected minimal number of cells to be searched according to M . More simply, we can state that if

$$H = \overline{H} - x$$

then

$$M = \frac{\overline{M}}{2^x}$$

Solving each equation for x and equating the solutions gives

$$x = \overline{H} - H = \log_2 \overline{M} - \log M$$

and solving for $\log_2 M$

$$\log_2 M = \log_2 \overline{M} - \overline{H} + H \tag{11}$$

Since

$$H = -\log_2 \frac{1}{N} = \log_2 N$$

and

$$\log_2 \overline{M} = \log_2 \frac{N}{2} = \log N - 1$$

substituting these in (11) yields

$$\log_2 M = H - 1 \tag{12}$$

or $$M = 2^{H-1} \tag{13}$$

Expressing H in terms of the original density distribution (the output of an ASWIPS) gives:

$$H = -\sum_i \frac{d_i}{K} \log \frac{d_i}{K}$$

and substituting in (12) to get $\log_2 M$,

$$\log_2 M = \log K - 1 - \frac{1}{K} \sum_i d_i \log_2 d_i \tag{14}$$

which can be regarded as the final computational formula for M. One advantage of using this relation to find M is that the need to actually determine the rank order of the d_i (necessary to the direct computation of M) is eliminated.

The relationship allows expression of the amount of information implied by a given density distribution in terms of H or M, depending on their respective meaningfulness in terms of the problem at hand. The following figure is a graphical representation of the relation.
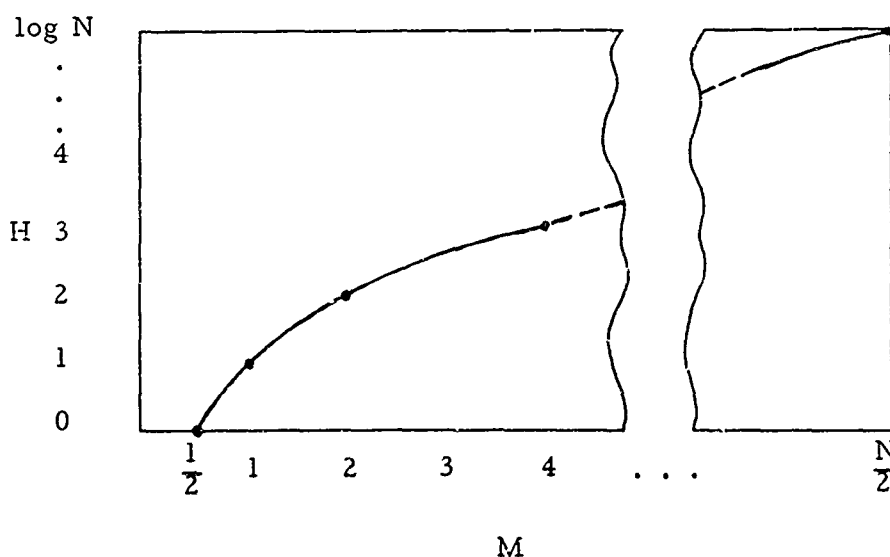
B.   The Comparison of Competing ASWIPS Designs

The information measure M developed in the preceding section is
based on the notion of "expected densities" (d_i) , which implies the notion
of "probability." We can reason that if the densities are based upon valid
probabilities, then M is a valid measure of the information output by the
ASWIPS. However, since the original purpose was to measure the output
of any ASWIPS, and since there is no reason to think that all ASWIPS
will be based on valid probabilities, the measure procedure must be further
developed.

Some ASWIPS, for example, may consist entirely of manual heuristic
procedures; hence, any probability densities derived or implied by their
output at any one time can at best be labeled "subjective probabilities."
At a more advanced stage an ASWIPS may employ probabilities based on
relative frequencies, in which case we have "probability estimates." Even
the most sophisticated systems of the future will undoubtedly make use of
a mixed bag of subjective probabilities, probability estimates, and assumed
probabilities before arriving at the final output density distribution.

When any of these systems is functioning in real time against an
actual enemy, there is no recourse but to assume the d_i outputs are the
real expectations, and therefore that the M or H measure discussed in
the preceding section reflects the correct amount of information obtained by
the system. Let us denote this type of measure as a "type A" score for the
given ASWIPS.

In the system design and test stage, however, there are more alter-
natives. Besides the d_i output by a system, there is available data con-
cerning the "actual" locations of the simulated enemy submarines. Hence,
there should be some systematic procedure for assessing the validity of
any system's set of d_i by comparison with the d_i of a superior system
(i.e., a system with more information available, such as the post-exercise
analysis team equipped with track charts prepared by the simulated enemy
submarines).

Let us denote the superior system, whatever the basis of its super-
iority, as the "comparison system." There is no requirement for the
comparison system to possess an absolute degree of accuracy of

submarine location. (For example, the post-exercise team cannot eliminate the uncertainties associated with various types of navigation errors.) The only essential feature is that the judges assessing a given ASWIPS have more confidence in the d_i set of the comparison system than in the d_i set of the given ASWIPS. By analogy, the comparison system is the standard yardstick.

We are now in a position to define the "type B score" for an ASWIPS output--one in which there is reference to a comparison system. We can do this with a slight change in the definition of M, i.e., by letting M' represent the expected number of cells containing all K submarines according to the ASWIPS ranking and the comparison system densities.

The difference between this notion and that for M is that, for M, the ranking and the densities are determined by the same system. But the difference can be seen more clearly by regarding the cells to be ranked according to the d_i set output by the ASWIPS, and the expected proportion of the K submarines in each cell to be given by the d_i set output by the comparison system. M' is then the mean of this resulting distribution.

For example, suppose that $N = 4$, $K = 2$, and the d_i's output and the resulting p_i are:

ASWIPS

d_i			p_i	
1	2		1	2
.5	.1		.25	.05
3	4		3	4
.4	1.0		.20	.5

Comparison
System

d_i			p_i	
1	2		1	2
.8	1		.4	.5
3	4		3	4
.2	0		.1	0

Then the density distribution for which we seek the mean is given by:
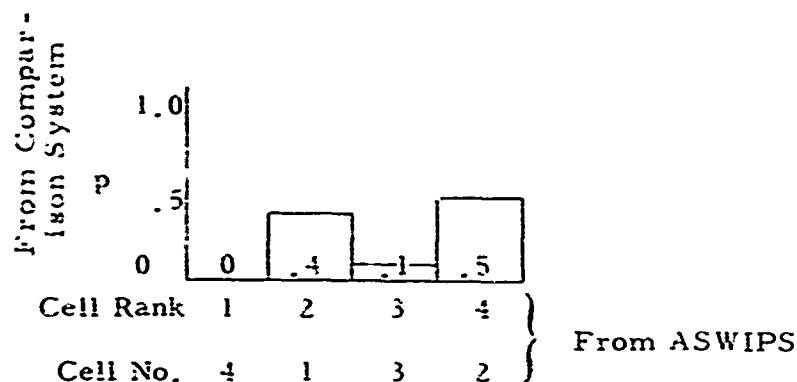
and $M' = \frac{1}{2} \times 0 \times 1 \frac{1}{2} \times .4 \div 2 \frac{1}{2} \times .1 \div 3 \frac{1}{2} \times .5 = 2.6$

This procedure includes the measurement of systems which may perform, so to speak, "worse then chance," since the mean M' may take on any value below the limit $N - 1/2$. On the other hand, M may not exceed $N/2$. This is perfectly reasonable, since there is no a priori reason to suppose that one might not encounter systems whose procedures, if followed, would lead to results poorer than those of chance alone. We can think of these possible systems as "perverse," in the sense that, while they possess information, this information is used (the ranking process) to "conceal" the submarines. The comparison measure M' should uncover such anomalies automatically.

The amount of information, in terms of H, that is possessed by such a "perverse" system may be found by:

$$H = \log_2 (N - M') + 1 \qquad (15)$$

where $M' \leq N/2$.

To summarize the procedure for measuring the adequacy of two or more competing ASWIPS at a given time (trial), the following steps are recommended.

1. Provide all systems with identical data from the "data producing" systems.

2. Obtain the density list, d_i, prepared or implied by each ASWIPS.

3. Obtain the M' score of each ASWIPS for this time using the same "comparison system" density list.

4. Find the average or mean M' across all times (trials) for each ASWIPS. Use this average M' as the basis for selecting the best ASWIPS logic. The minimum average M' should belong to the "best" system.

III. AN ASWIPS SYSTEM DESIGN

A. Introduction

This section presents an outline of interlocking ideas which could provide the basis for designing an ASWIPS that would yield near optimal scores as measured by M'.

The system output is a single display showing:

- Density of expected submarines across the command sector
- Current probable tracks of submarines
- Current and recent datums (positive sensor contacts)
- Current information score of type A (i.e., the M score)
- Current estimate of K for the sector

The system design is based upon three main methods. The first is the use of Bayes Theorem for all cases in which the probability of occurrence of an event must be deduced from the occurrence of another event. The second method is to use the concept of diffusion to represent the uncertainties of submarine location due to the probable movements of the submarines in the absence of sensor data. The concept of diffusion is borrowed from the physics problem describing Brownian Motion, the random motion of small particles suspended in a fluid. The concept is also related to the mathematical notion of "random walks." The third technique, probable track analysis, not only yields the output "probable tracks" and the probabilities associated with each link of the track, which are of interest in themselves to the command, but is an integral part of three other processes of the system:

- Determination of the probability that a given possible submarine is diffused later about datum
- Modification of K for the sector from previous estimates
- Modification of the transition matrix used in the diffusion process

The computer program embodying this procedure would be operated, or "turned over" for one cycle, either: (1) upon receipt of a new datum (i.e., positive contact data); or (2) upon the passage of an

arbitrary time frame (chosen by the command) since the last turnover--
whichever occurs first. In either case, whenever the program is oper-
ated, certain data concerning the sensor systems operating in the
command sector must be input, whether or not there have been any posi-
tive contacts. These are specified in subsection III. D below and referred
to where necessary in the other sections.

The display changes only upon the operation of the program.

B.   Matrix Organization

The command sector, with area A, is divided into N type B cells,
each with area b = A/N . These cells may be either squares, triangles,
or hexagonals, the only unit forms which when replicated will fill a plane
with no gaps. Each of these forms has certain advantages and disadvantages
which are of interest for specific systems but not central to this discussion.
The system needs one such matrix to contain the output densities and another
of the same size, called the "artificial datum," to contain the diffused den-
sities not associated with any currently maintained datums.

A third matrix is required to represent the transition probabilities
used in the diffusion process. The structure and size of this third
matrix is discussed later; however, there must exist a relation between
its cells and the cells of the other two matrices. Separate matrices,
but of smaller size, using the same cell area B, are used to carry the
diffused densities associated with each currently maintained datum, one
for each such datum.

Finally, a "master sector matrix" shows the spatial relationships
between all sectors in the world which contain submarines that could
affect operations in this sector during the life of the program operation.
This matrix has several purposes. One is to contain current estimates
of the K for each sector, as a bookkeeping device. The matrix also
indicates transition probabilities between whole sectors. Certain arti-
ficial sectors can be used to represent production of new submarines
and in/out transitions for overhaul, maintenance, decommissioning,
and losses. These remarks are useful primarily for applications to
large sectors. For application of this system approach by individual
units of HUK commands, the master sector matrix need only represent

nearby sectors with submarines that could transit in or out of the command area during the time of operational interest.

The primary reasons for requiring the master sector matrix are:

- To prevent loss of densities in the command sector when these diffuse across the sector boundaries; the sum of the densities across all cells of the sector must be K .

- To keep account of changes in the estimate of K due to diffusion in and out of the sector.

- To keep account of changes in the estimate of K when contacts in adjacent sectors are correlated (i.e., when probable tracks cross sector boundaries).

The probabilities to be carried in the transition matrices are conditional upon the passage of a unit time, ΔT . That is, the probability of transiting from cell x to a specific adjacent cell during the time ΔT , given that the submarine was in cell x at the beginning of the ΔT time frame, is called a transition probability.

Suppose that each cell has e adjacent cells (depending on the shape and arrangement of cells). Then the transition matrix shows, for each cell x, the probability of transiting to each of the e adjacent cells and the probability of remaining within cell x, given that a submarine is in cell x at the beginning of the ΔT frame. This requires $e + 1$ numbers for each cell x. If the transition matrix is limited to this form, there are certain disadvantages (which, however, may not matter in certain applications): namely, that certain types of diffusion behavior, such as "routings" and "patrol areas" cannot be represented.

"Routings" are recurrent pathways used for transiting from one area to another. Their discovery and representation could be very important in the predicting process. "Patrol areas" are areas in which transiting behavior is absent. Once in such an area, the submarine's movement may be more or less at random. Again, the discovery and representation of this fact by the system could be important.

These phenomena and their distinctions cannot be represented in the above-defined transition matrix because the transitions there are

independent of the direction of entry into cell x.   In the case of application of this system by unit commands or HUK commands seeking submarines in a local area on a detailed time basis and for which evasion or attack by the submarine vis-a-vis his knowledge of the ASW forces dominates his long term transit or patrol behavior, all indications are that the above transition matrix formulation is sufficient.

With this exception the transition matrix should contain provision for transits to each adjacent cell or for remaining in cell x, given original entry by each of the adjacent cells.

The following diagram may clarify this. Suppose that each cell of the sector matrix is adjacent to at most six cells. Then for each cell x, the following table is required to contain the transition probabilities.

Given That Original
Entry Was From
Adjacent Cell:

	Cells Transited To During ΔT						
	x	1	2	3	4	5	6
1							
2							
3							
4							
5							
6							

The sum of the probabilities in each row is required to total to 1. The probabilities themselves may be:

- Assumptions
- Empirically found relative frequencies
- A mixture of the above two types. This is, in fact, the recommended procedure, as explained below.

When no data is available, as we may assume is the case when the system is initially applied, the probabilities should be subjective ones based on expert operational opinion. If probable track links between two datums are formed by the system, then all cell transits (from and to) can be used to modify the corresponding probabilities. This should be done by assigning weights to the previous probabilities relative to the one transit observed, as follows.

Let $\hat{p}_{jk}$ represent the previous probabilities of a transit to j in ΔT, given original entry from k, for $k = 1, 2, \ldots 6$ and $j = 1, 2, \ldots 7$, with $j = 1$ representing the case where the submarine remains in cell x.

Let O_{jk} represent the observation of a transit to j, given original entry from k, for $j = 1, 2, \ldots 7$. $O_{jk} = 1$ if the transit was to j; $O_{jk} = 0$ otherwise.

Then the new transition probability p_{jk} can be found by

$$p_{jk} = \hat{p}_{jk}\, W + O_{jk}\, (1 - W) \tag{16}$$

for fixed k observed and $j = 1, 2, \ldots 7$, where W is the weighting factor and $0 < W < 1$.

A large W means that more weight is given to past observations and assumptions, and this makes the system more conservative to change. If W is made smaller, the system becomes more adaptable to the current situation. Setting W (or W's if the user/designer wishes to distinguish this factor by areas) is a matter of judgment for the user/designer.

It should be pointed out that a special significance attaches to the probability that the submarine remains in cell x (column 1 of the previous figure). This number, unlike others in the rows, controls the rate or speed of the diffusion process. The other probabilities affect only the direction of the flow. If the probability of remaining in the cell is close to 1, the diffusion is slow; the closer to 0, the faster the diffusion. This should especially be kept in mind when establishing initial assumed values. It will be automatically taken care of in the updating process.

Matrix organization and operations in a computer directly affect the pragmatic efficiency, cost, and sometimes even the feasibility of an operational system. Suggestions in this area and some recommended algorithms will be found in a later subsection.

C.   Establishing K

As explained above, the system keeps account of the expected
number of submarines, K , for each sector in the master sector matrix.
The general procedure of the system is:

- To establish initial values of K by assumption, or by
  judgment based on intelligence data.
- To modify the current K's by sector automatically as the
  evidence derived from sensors and diffusion processes
  warrants.
- To modify the current K's by sector if new intelligence
  not derived from operational sensors warrants such a
  change.

Besides the K's , initial assumptions on intelligence data should
be used to establish the transition probabilities between sectors. As
was explained in the previous subsection, one of these probabilities
(the probability of remaining in the cell or sector) governs the rate of
change or speed of flow between cells or, in this case, between sectors.
The ΔT for the sector transition matrix should be identical to the ΔT
used in the cell transition matrices.

Artificial sectors can be used to represent enemy ports, main-
tenance and production facilities, and "sinks." In this way, the rates
of flow from or to these artificial sectors can be used to control the
rates of change to K due seperately to:

- Production of new submarines
- Overhauls
- Port stays and maintenance
- Losses
- Decommissionings
- All other causes

If desirable, these rates can be made time dependent, increasing,
decreasing, or cyclic.

If the time dependence is desirable for the application, then the
transition matrix can be changed by new inputs from time to time, or
forecasts can be inserted initially and a requirement laid on the program

to look up the appropriate rates as a function of current time (which would be an input for each turnover of the program). The methods by which the K's are altered as a result of sensor contacts and the diffusion process are explained in detail in a later subsection.

D. Contact Inputs

One of the inputs to the system operation is a description of positive sensor contacts, hereafter called "datums." In general, the following items are required to describe a datum:

1. Time of contact. If contact was being maintained at the time the message was forwarded, then this time is the time of message; if contact was lost when the message was sent, this time should be the time contact was lost.

2. Datum location. This would normally be a latitude-longitude specification of the center of the datum area (see below).

3. Datum area. This is a description of the shape and extent of the area, in relation to the datum location, which includes the contact (possible submarine) with certainty. In some cases parameters enabling this description may be actually forwarded. In other cases the shape and extent of the area may have to be determined by procedures and parameters stored in the ASWIPS computer. In either case, it is assumed in the system design described hereafter that the datum area is available, whether by direct input or prior processing.

4. Sensor/platform identification. This enables the system to "table look-up" certain parameters needed in relation to the type of sensor system (to be defined in more detail below). There is, in general, no limitation in the system design on types of sensor/platform combinations.

5. Classification. This heading refers to the types of possible causes of contact (i.e., hypotheses), with associated indices for validity of confidence corresponding to each given hypothesis. Normally, a sensor will report only a contact it

classifies as a possible submarine. However, if the sensor
so reports and later changes its classification, it is impor-
tant that the new classification also be input and that a cross-
reference be made to the previous datum.

The index of validity of confidence associated with the hypo-
thesis may be either a true probability estimate determined
by a sensor system or, as is more likely, an arbitrary scale
index. In the latter case, there is a requirement for the
system computer, by preprocessing, to assign a probability
estimate to the hypotheses as a function of the scale index
and the sensor identification, based upon either empirical
past relative frequencies or subjective probabilities from
expert opinion. The hypothesis may include more than one
type of enemy submarine. This is desirable when it is
formed from actual evidence, as the system will consider
data on different types in determining the probability of
track linkage between contacts.

In the next subsection, the classification process is exam-
ined in more detail, primarily because the ASWIPS defined
here may be applied in situations where it is itself considered
as a "classifier" for reporting datums to other, possibly
higher level, commands.

E.   Datum Uncertainty Bookkeeping

The central process of the ASWIPS is the computation concerning
the probability of "links" between datums. A link is defined as a straight
line between the centers of mass of two datum areas. The line repre-
sents a possible "average track" and implies, by the associated pro-
bability, the likelihood that these two datums are from an identical
enemy submarine. The direction of travel is implied by the associated
datum times (i.e., from the earlier to the later datum).

Suppose that PL_{jk} represents the probability of a link going from
datum j to datum k. Then we define

$$PS_k = \sum_j PL_{jk} \tag{17}$$

which is to say that all probabilities of links coming to datum k sum to the probability that datum k is an enemy submarine, PS_k . Similarly, for the sum across all links going from datum j

$$\sum_k PL_{jk} = PS_j \qquad (18)$$

The PL_{jk} are a posteriori Bayesian probabilities of the hypothesis that datums j and k are the same enemy submarine. It follows that, for all jk , $PL_{jk} \geq 0$. The a priori probability of this hypothesis is defined as the sum of the datum j densities across the datum area of datum k of the time of datum k . By "datum j densities" is meant the probability density function representing the probability of possible submarine j across geographic space as of the time of occurrence of datum k .

This technique calls for maintaining at all times in the system an "artificial" datum which represents the expected submarine density across the command area of all submarines in the area not associated with currently maintained datums. Thus, each datum in the system will be "linked" both from and to this artificial datum. When the current oldest datum is to be dropped by the system either because the datum capacity of the computer is reached or because the absolute age of the datum makes it unimportant, the probability density function for this oldest datum is added to the artificial datum. Initially, the only datum may be the "artificial" datum, and its distribution will be the a priori submarine density at the commencement of operations.

The system bookkeeping for link probabilities can best be illustrated by the table in Exhibit 1, which contains example probabilities for a case where K = 2 . The table is indexed by j vertically, representing the "from" datums, while the columns are indexed by k for the "to" datums. The datums are labeled as "out" (to be explained subsequently), "art." (the artificial datum), and by arbitrary identification labels L_j (where the oldest datum is represented by L_1 the next oldest by L_2 , etc.).

EXHIBIT 1 - TABLE FOR BOOKKEEPING PROBABILITIES OF LINKS

"To" Datum

"From" Datum	j Index	Datum Identification	k Index PS	1 3.0	2 2.0	3 .9	4 .8	5 1.0	6 .5
	1	"Out"	3.0	3.0	0	0	0	0	0
	2	"Art."	2.0	0	.300	.9	.8	0	0
	3	L_1	.9	0	.075	X	0	.75	.075
	4	L_2	.8	0	.525		X	.25	.925
	5	L_3	1.0	0	.600			X	.400
	6	L_4	.5	0	.500				X

The cell entries represent the probabilities of a link from the jth
to the k datum. Notice that the sum of the PL's across a row or col-
umn equals PS_j , the probability that datum j is a submarine, except
for the artificial datum, in which case it sums to K , the current
expected number of submarines in the command sector. When the old-
est datum (index 3) is to be merged back with the artificial datum, the
bookkeeping rule is to add row 3 to the "art." row and shift all higher
indexed rows and columns down one index, thus destroying row and
column 3.

The quantity in the "art." column represents, for datums j = 3,
4, ..., the expected number of submarines diffused about datum j as
of the time of the latest datum. This follows from the fact that datum
j is linked to other later datums with probability $PS_j - PS_{j,2}$, and
hence that this number of expected submarines is associated with the
diffused densities about these later linked datums. The cell represent-
ing the linkage from the "art." datum to the "art." datum (cell 2,2)
represents the expected number of submarines in the command sector
currently "undetected" (i.e., not associated with current datums).

The "out" datum is used to represent linkages of datums inside
the command sector with datums in adjacent sectors, either incoming
or outgoing. The procedure for computing the probabilities of such
links is different from the normal procedure to be described in sub-
section III. F below for linking datums totally within the command sector.

The sum of the "out" column and row should equal the current sum
of the K's for all adjacent sectors (3 in the example). Upon receipt of
a new datum, say n , a new column vector of PS_{jn} is computed by
methods discussed in the next subsection. The bookkeeping procedure
then calls for replacing the values in the artificial column according to
the equation

$$PL_{j,2} \Rightarrow PL_{j2} - PL_{jn} \tag{19}$$

for j = 1, 2, ...n - 1 , and setting

$$PL_{n2} = PS_n \tag{20}$$

This corresponds to reducing the uncertainty remaining in spaces relative to the linked "from" datums, since we have evidence of linkage. If this causes any of the PL_{j2} values to become negative, we have an indication that certain assumptions have been violated. The significance of the violation will depend upon the application, and consequent action programmed for this eventuality should therefore remain with the applications analyst and his specific problem. However, for applications calling for an initial estimate or assumption of K, the following algorithm is applicable.

If $PL_{22} < \epsilon$ (i.e., the expected number of undetected submarines is less than the arbitrary small positive quantity), increase K to

$$K \Rightarrow K - PL_{22} - \epsilon \qquad (21)$$

and reset

$$PL_{22} \Rightarrow \epsilon \qquad (22)$$

By this means, K will be automatically increased if the evidence warrants. If a maximum value of K is known with certainty, then when K exceeds this maximum the status of the system reverts to that of having violated some system assumption, and the foregoing remarks apply. For the case of uncertain K, PL_{22} is maintained greater than or equal to ϵ to ensure a positive a priori probability of linkage to the possibly undetected number of submarines.

F.   Probability of Track Links

Determination of the probabilities of links, PL_{jk}, is accomplished by substitution in Bayes Theorem, which follows:

$$P(H_i/D_g) = \frac{P(H_i)\ P(D_g/H_i)}{\sum_i \hat{P}(H_i)\ P(D_g/H_i)} \qquad (23)$$

where

$P(H_i/D_g) \equiv$ the posterior probability of hypothesis i, given the occurrence of datum g

$\hat{P}(H_i)$ $\equiv$ the a priori probability of hypothesis i

$P(D_g/H_i)$ $\equiv$ the probability of datum g , given the occurrence (or truth of) the ith hypothesis

and in which the following constraints are implied:

$$\sum_i P(H_i) = 1 \qquad\qquad (24)$$

$$\sum_i P(H_i/D_g) = 1, \text{ for all } g \qquad\qquad (25)$$

$$\sum_g P(D_g/H_i) = 1, \text{ for all } i \qquad\qquad (26)$$

Upon the receipt of a new datum, the ASWIPS treats each current datum from $j = 2$ through n of the "link table" as a hypothesis indexed by $i = 1, 2, \ldots (n - 1)$. In addition, each type of false contact, $f = 1, 2, \ldots m$, for which the ASWIPS maintains a density function is treated as a hypothesis indexed by $i = n, n + 1, n + 2, \ldots (n - 1 + m)$. In each case, $\hat{P}(H_i)$ is set equal to the sum of the densities of the ith hypothesis across the datum area.

The datum index g specifies the type of datum "report" received or observed. This enables a table look-up of the prestored $P(D_g/H_i)$, which is, in part, a function of the sensor system which observed the datum.

The computation of the posterior probabilities, $P(H_i/D_g)$, by Bayes Theorem is then performed for $i = 1, 2, \ldots (n + m)$. These results are stored in the link table, column $n + 1$, for $i = 1, 2, \ldots (n - 1)$ stored by index $j = 2, 3, \ldots n$.

The probability that the new datum is a submarine is computed by

$$PS_{n + 1} = \sum_{i = 2}^{n} P(H_i/D_g) \qquad\qquad (27)$$

and stored in the appropriate place in the link table.

The above procedure holds only when the new datum as well as all
past datums considered as possible links are within the command sector.
If any of these datums are in adjacent sectors, a slightly modified pro-
cedure must be employed, since datum density distributions are main-
tained only across spaces inside the command sector.

The correlation of linkage of contact datums across sector boun-
daries is an important consideration for the ASWIPS, primarily because
this is one of the three sources of information enabling a modification of
the current estimate of K for the command sector. K should be increased
or decreased by the link probability, according to the direction of the
link (passing out or coming into the sector). Another source or method
of determining the a priori probabilities for the linkage is required, but
once they are determined, the above-described Bayes procedure is
employed.

Since we may expect such linkage to be relatively rare (how rare
is partly a function of the sector size), we can tolerate a cruder estima-
tion process without expecting much overall loss of information. A
recommended procedure is to predetermine a set of curves (or functions)
for computer storage, giving the a priori probabilities desired as a
function of:

- Distance between datums
- Time difference between datums
- Datum area
- Probability that the "from" datum is a submarine
- Average speed of advance of the possible submarine assoc-
  iated with the "from" datum
- Sensor action (if any) in the vicinity of the two datums

The predetermination may be accomplished by a Monte Carlo process.
Since this is a complicated subject of analysis in its own right, it will
not be explored further in this document. It should be sufficient here
to point out how the results of such a model are incorporated into the
ASWIPS and used.

Once the ASWIPS has determined the required a priori probabilities
by table look-up and/or computations involving links across boundaries,

and has performed the Bayesian computation of the posterior probabilities of linkage, the results are stored in the link table as described below.

Case 1:   The new datum lies in an adjacent sector, and all other datums considered (those already specified by the link table) are inside the sector.

in this case, the results (the column vector containing probabilities of links between the old datums and the new datum) are added to the results already contained (if any) in the column labeled "out" (outside the sector). It should be pointed out that in this case the artificial datum should not be included in the set of old datums linked to the new datum. The possibility of this kind of linkage is treated in a different manner below under the subject of diffusion outflow and inflow across sector boundaries.

Case 2:   The new datum lies inside the command sector, and at least one "from" datum considered is located in an adjacent sector.

In this case, the results (the column vector) are stored in the next available empty column. The sum of link probabilities with all "from" datums outside the sector is stored in the "out" position of the column vector (i.e., the position with index $j = 1$).

In either of the above two cases, the link table update procedure which follows the insertion of new column information will automatically adjust the current estimate of K , up or down, in accordance with the linkage direction and amount as this is reflected by the changes in the "out" row and column.

G.   Diffusion and Negative Detection Processes

For each datum currently represented in the link table (including the artificial datum but excepting the "out" datum), the ASWIPS will maintain a separate diffusion matrix. These matrices have cell size and arrangement identical to the display matrix, but are spatially oriented relative to the center of gravity of the original datum area input.

The cells contain the probability that the contact object is in the cell at a given time. The sum of these probabilities equals 1; hence, these probabilities are conditional upon the contact's being an enemy submarine. In this way the diffusion process need not be altered as the evaluation of the datum (as represented by the probability of its being a

submarine, PS) undergoes changes due to the linkage process.

When the ASWIPS receives a new update time (i.e., either the time of a new datum input, or an arbitrary time input), the program updates the diffusion matrix for each datum, considering both:

- The submarine movement probabilities as represented in the transition matrix.

- The alteration of the submarine location probabilities as a function of any "negative" sensor action. If a sensor has the potential of a submarine detection across a specific area (called the "look" area) at a given small time span, and if there are no contacts generated during this span, this is called a negative sensor action. In order not to lose the information implied by this action, the ASWIPS evaluates the impact of negative sensor action and integrates it with all other information sources.

The general algorithm for this subsection, which treats both of the above effects, is as follows, for each datum individually.

1. The diffusion matrix is "multiplied" times the transition matrix to obtain the effect of a one-step transition, corresponding to a passage of time ΔT. This multiplication process is defined in subsection III.I below.

2. The cells of this new diffusion matrix which lie within any sensor "look" area at this time have their probabilities reduced in correspondence with the given sensor "probability of a submarine given no contact." (This procedure is defined in detail in subsection III.H, although subsection III.H considers a more general problem than that posed here. In the case of this application to subsection III.H, the datum type refers to "no contact," and the datum area is the no-contact area.) This process is repeated for each sensor. If more than one sensor "looks" at a given cell, they are assumed to be independent looks (in the statistical sense of the word) and the same algorithm is applied in succession, the nth sensor being applied to the probability

of a submarine in the cell resulting from the n - 1 sensor. The total <u>reduction</u> in probability across all such cells within sensor look areas is then accumulated, say R .

3.   To maintain the constraint that the summation of probabilities across all cells in the diffusion matrix equals 1, the probabilities of all cells outside any look areas are increased. The amount of increase is proportionate to the cell's probability (p) relative to the sum across all other cells outside the look areas, say S . In other words, p is replaced as follows

$$p \rightarrow p + \frac{pR}{S} \tag{28}$$

since no evidence has been received concerning the relative differences between probabilities in these cells.

4.   Steps 1 through 3 are repeated until the matrix "as of" time equals the given update time. In each iteration, movable sensors are advanced along their tracks (which are input) in accordance with their input speed for a time ΔT .

H.   Posterior Probabilities Given Sensor Data

By definition, as long as a sensor is operating (looking) at spaces within the command sector, there is a look region specified at any arbitrary time. Further, at any arbitrary time the look region is subdivided into at most two mutually exclusive regions, a contact region and a no-contact region. Also by definition, any cell lying within a contact region is a cell within the datum area for all datum types g , implying a contact. Any cell lying within a no-contact region is a cell within the datum area for the one datum type implied by no contact.

Consider a cell c lying totally within the datum area. Let

$\hat{P}_c(S)$ ≡   the <u>a priori</u> probability that the possible submarine associated with a given datum j of the link table is in cell c at a given time, assuming that the original datum was a submarine (taken from the diffusion matrix j)

Let

$\widehat{P}(S)$ $\equiv$ the <u>a priori</u> probability that submarine j is within the datum area

$$\widehat{P}(S) = \sum_{\substack{c \in \\ \text{datum} \\ \text{area}}} \widehat{p}_c(S) \qquad (29)$$

This represents a summation of all such cells c included within the datum area.

Let

$\widehat{P}(\bar{S})$ $\equiv$ the <u>a priori</u> probability that the submarine j is not within the datum area

Then

$$\widehat{P}(\bar{S}) = 1 - \widehat{P}(S)$$

Let

$P(D_g/S)$ $\equiv$ the probability of obtaining datum g , given a submarine within the datum area

$P(D_g/\bar{S})$ $\equiv$ the probability of obtaining datum type g , given no submarine in the datum area

$P(S/D_g)$ $\equiv$ the posterior probability of a submarine j within the datum area, given datum type g

Then by Bayes Theorem,

$$P(S/D_g) = \frac{\widehat{P}(S) \cdot P(D_g/S)}{P(S)P(D_g/S) + \widehat{P}(\bar{S}) \cdot P(D_g/\bar{S})} \qquad (30)$$

Let

$p_c(S/D_g)$ $\equiv$ the posterior probability of submarine j in cell c , given datum type g

If we assume the posterior distribution across cells in the datum area to be relatively unchanged by the sensor action, then

$$p_c(S/D_g) = \frac{p_c(S) \cdot P(S/D_g)}{\hat{P}(S)} \qquad (31)$$

Substituting the righthand side of equation (30) for $P(S/D_g)$ above gives

$$p_c(S/D_g) = \frac{\hat{p}_c(S)P(D_g/S)}{\hat{P}(S)P(D_g/S) + \hat{P}(\bar{S})P(D_g/\bar{S})} \qquad (32)$$

If the boundary of the datum area passes through a cell, then for computational purposes temporarily redefine the cell c to be that sub-area within the original cell c' , and let

$$\hat{p}_c(S) = \hat{p}_{c'}(S) \cdot \frac{\text{area of } c}{\text{area of } c'} \qquad (33)$$

After finding the posterior probabilities of all sub-cells of the original cell c' , sum these to obtain the posterior probability for c' .

I. Computation for One Transition Step of Diffusion Matrices

Suppose for each cell c there are at most e adjacent cells labeled x = 1, 2, ... e .
Let

p_{xc} $\equiv$ the probability that possible submarine j (of link table) is in cell c <u>and</u> entered from cell x , given that datum is a submarine

Then

$$\sum_x p_{xc} = p_c(S) \qquad (34)$$

where $p_c(S)$ is as defined in subsection III.H.
Let

$p_{xc}^{\not/}$ $\equiv$ the probability that the submarine will remain in cell c after ΔT, given that the submarine is in cell c and originally entered from x

$P_{x\overrightarrow{c}}$ $\equiv$ the probability that the submarine will transit from cell c during ΔT, given that the submarine is in cell c and originally entered from cell x

Then

$$P_{x\overrightarrow{c}} = 1 - P_{x\overset{+}{c}} \tag{35}$$

Let

$P_{x\overrightarrow{c}y}$ $\equiv$ the probability that the submarine will transit to adjacent cell y during ΔT, given that the submarine is in cell c and originally entered from cell x

These are subject to the constraint

$$\sum_{y=1}^{y=e} P_{xcy} = P_{x\overrightarrow{c}} \tag{36}$$

Suppose that the cells adjacent to a given cell x adjacent to c are indexed by z . Then after one transition step (corresponding to the passage of time ΔT), P_{xc} is replaced as follows:

$$P_{xc} \Rightarrow P_{xc} \cdot P_{x\overset{+}{c}} + \sum_{z} P_{zx} \cdot P_{z\overrightarrow{x}c} \tag{37}$$

Applying equation (34) gives the updated $p_c(S)$.

The reader is reminded that ΔT and the cell size are specified such that a submarine may transit to at most one adjacent cell during the time ΔT.

J.   The Master Sector Matrix

This matrix, defined previously, contains a "logical" cell for each sector to represent the K (expected number of submarines) of each sector. Associated with this matrix is a transition matrix showing,

for each sector, the probability of transiting to an adjacent sector and
the probability of remaining in the sector, based on the same ΔT
passage of time used for the command sector transition matrix, and
given that the submarine is in the given sector. The transition pro-
babilities from this command sector are variable, unlike those for all
other sectors, and are found by accumulating the diffusion overflow
across adjacent sector boundaries when "diffusing" each datum in the
sector.

The master sector matrix diffusion algorithm is logically identi-
cal to that described for diffusing the sector's datum distributions,
except that negative sensor data is not treated for this scale. The pur-
pose in this operation is to treat in a systematic fashion possible
changes to the K's , based on worldwide intelligence data of both a
strategic and tactical character. For systems operating in other sec-
tors (especially, for example, "counting barriers") and capable of
providing frequent new estimates of K for their sectors, the master
sector matrix is also the modus operandi for receiving and using these
new K estimates if they are received by this sector command.

K.   System Flow and Summary

The foregoing discussion of system concepts was without regard
to the sequence of system operation. For this reason the system oper-
ation and sequencing will be done here in summary fashion, referencing
the already established detailed subroutines.

Upon receipt of a new datum or "update time" (which defines the
next "operation time"):

1.   Obtain all ASW platform/sensor movement data for times
      since the last operation through this operation time.
      Enter new datum information (if any) in the link table.

2.   Diffuse all datums in the link table from the previous oper-
      ation time to the current operation time, simultaneously
      considering the effect of negative sensor data.

      a.   Accumulate all diffusion overflow from the command
            sector to each adjacent sector. Place this overflow
            amount in the master sector matrix.

b. Obtain the a priori probabilities of a link between each datum in the link table and the new datum (if any) from the final diffusion matrix of each datum.

c. Set up a new diffusion matrix for the new datum (if any) with a density distribution across its datum area.

d. Compute the a priori probabilities of a link across any sector boundary, making use of the "out" summary datum.

3. Compute the posterior probabilities of track links from all of the old datums to the new datum (if any), and enter these in the link table.

4. Diffuse the master sector matrix from the previous to the present operation time to obtain:

a. Diffusion inflows to this sector from adjacent sectors; enter this amount in the link table from the "out" row to the "art." column.

b. Changes to K for all sectors except this command sector.

5. Operate the link table update algorithm to obtain the new datum sum of densities, PL_{j2}, for each datum and the new estimate of K for this command sector.

6. Accumulate the sum of the diffused densities across all datums, each multiplied by the datum's PL_{j2} to obtain the total sector density distribution. Output for display.

7. Add the new track links (if any) that have link probabilities exceeding the threshold (a parameter) to the display of probable tracks.

8. Use the new track links (if any) to modify the transition matrix.

Upon receipt of new estimates for K from other sectors or new transition probability estimates, the ASWIPS system simply incorporates these new estimates in the appropriate matrices, and performs no additional processes.

IV. APPLICATIONS

This section examines certain additional problems which will arise in applying the ASWIPS design described in Section III to a specific ASW problem.

A. Differentiating Submarine Types

In Section III, the problem was stated in terms of locating enemy submarines, without regard to type of enemy submarines. This was done primarily to avoid obscuring essential system concepts. In addition, for some potential systems there may be no requirement for differentiating by type or there may be insufficient computer capacity or computer time available to carry through the differentiation by type. For such cases, this subsection may be ignored. It must be pointed out, however, that all of the probabilities defined in Section III for this situation should be understood as averages across all types of enemy submarines. This applies to both the diffusion transition matrices and the sensor descriptive $P(D_g/H_i)$, as well as the a priori density distribution of the "artificial" datum and the sector K's of the "master sector matrix."

To add the capability of processing ASW data by type and outputting, on demand, a density map by type, it is sufficient to add to the system described in Section III the following considerations.

1.   A list of the type distinctions of interest to the command is made.

2.   A separate diffusion transition matrix is maintained for each such type.

3.   A separate diffusion matrix by type is maintained for each datum in the link table.

4.   A separate master sector matrix of K's is maintained for each type.

5.   A separate master sector transition matrix is maintained for each type.

6.   The rows of the link table are expanded to include a separate row for each type for each datum. The columns of this table remain as defined.

7.   The hypotheses set H_i used in applying Bayes Theorem is expanded such that, instead of one enemy submarine hypothesis for each datum, there is now one for each type under each datum. The non-enemy submarine hypotheses are unchanged in concept.

8.   The set of sensor descriptions, $P(D_g/H_i)$, is expanded in accordance with the expansion of the set of H_i as in paragraph 7 above.

9.   The set of possible D_g is formulated to include data which permit differentiation by type (i.e., there is a shift of emphasis here from data which discriminate only between enemy submarines and non-enemy submarine objects to data categories also capable of discriminating between types). (See subsection IV.B below for an expanded explanation of this requirement.)

10.   The posterior Bayesian probabilities for hypotheses representing past datum-type combinations are inserted in the column of the link table representing the new datum.

11.   The "PS" column (probability of a submarine) in the link table is relabeled "PST" (probability of type submarine). For a new datum for which the posterior probabilities have been computed, the "PTS" entries represent the sum across past datums for each type of the probabilities in the new datum column.

12.   The link probabilities for the new datum which are displayed in association with the "probable tracks" are found by summing across types for each past datum of the probabilities in the new datum column.

13.   The link table update algorithm is unchanged in terms of the constraints imposed on the rows and columns, although the rows now represent datum-type combinations.

14.   The display matrix for a given type is found by summing across all diffusion matrices for this type.

15.   A composite display matrix may be found by summing across all type display matrices.

16.   This system enables the command staff to selectively call up density displays by type or to call up a composite density display depending on their interest at the moment. In addition, the probable track display, although structurally identical to that formulated in

Section III, has a more "refined" set of link probabilities due to the correlation of links by type (e.g., datums with a high probability of representing different types of enemy submarines will be linked with a low probability, despite favorable time-distance relationships).

B. Sensor Submodels

This section is essentially an expanded explanation of the set of probabilities, $P(D_g/H_i)$, referred to in subsection IV.A above and throughout Section III. There should be a set of these quantities for each type of sensor system which may feed information to the ASWIPS. These are prestored in the computer for table look-up when needed. They are therefore fixed parameters which may be varied when and if better information on them becomes available.

The D_g are datum types or categories of sensor (usually sonar) signals. They may represent simple or complex combinations of subvariables concerned with the sensor system. These subvariables may be defined in terms of very "raw" data (i.e., a frequency spectrum analysis) or in terms of abstract classifications (e.g., the output of a classification process). The definitions of the D_g may also include non-sensor variables such as those pertaining to environment or location.

Ideally, the probabilities $P(D_g/H_i)$ would be empirical relative frequencies with a large data base. Lacking this data base, they may be set initially by expert judgment or exterior models and modified as data becomes available in a manner similar to that discussed for the submarine transition probabilities. For bookkeeping this process, it may be desirable to create a special subsystem of the ASWIPS.

The classification by this ASWIPS of a given datum in terms of the type submarine found in the link table might also serve as the basis for a set of D_g to be forwarded to a higher command. In this way, it is easy to conceive of a hierarchically related network of ASWIPS for the distant future.

C. Special Diffusion Submodels

Once the level and scale of a specific ASWIPS is fixed, it is possible to think of creating special submodels for various

generate the transition matrices, based on different assumptions about submarine movement or evasion policies. This idea is particularly applicable at the lowest command level (involving searches for one specific type submarine in an area close to datum). For example, a Monte Carlo type model could be designed to generate transition probabilities on the basis of "random walks." The parameters controlling the nature of the random walk could be made to represent a particular submarine evasion policy. Although this is a special field of investigation in its own right, the point is that the results of such studies could be integrated into certain ASWIPS and utilized in real time.

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Planning Research Corporation 1875 Connecticut Ave., N.W. Washington, D.C. 20009	UNCLASSIFIED
	2b GROUP

3. REPORT TITLE

System Design Considerations for a Bayesian Antisubmarine Warfare (ASW) Information Processor

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final

5. AUTHORS (First name, middle initial, last name)

Livingston Dodson

6. REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
April 1969	49	

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
Nonr 4994(00)	
b. PROJECT NO RF 118-98-02	PRC R-1319
c. NR 273-008	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12 SPONSORING MILITARY ACTIVITY
	Office of Naval Research Naval Analysis Programs Washington, D.C. 20360

13. ABSTRACT

This study continues the investigation of methods for applying Bayesian statistics to the problem of locating submarines from data made available to an ASW Command Center. The work performed during earlier phases of this project is documented in PRC reports: R-808, Bayesian Information Modeling in ASW, Feb. 1966; R-1001, Application of Bayes' Theorem to Predicting Submarine Locations (U), Feb. 1967 (Secret); and R-1133, Analysis of ASW Data Using Bayesian Techniques (U), Feb. 1968 (Secret).

In particular, this study investigates methods for:
- The simultaneous integration of data across an area containing possibly more than one, i.e., K, submarines.
- Determination of "probable tracks" and their "validity" in conjunction with processes which determine a submarine "density map" for display purposes,
- A generalization of the previous concept of submarine "diffusion" (to represent unknown but possible submarine movements) in order to bring the model closer to the complexity of real operations.
- A measurement of effectiveness of any combined ASW system, i.e., sensors plus data integration, concerning its localization of enemy submarines.

DD FORM 1473

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	AT	ROLE	AT	ROLE	AT
ASW, Bayesian Information Processor, command and control						